A Networks Perspective on Temporary Migration and Risk-Sharing*

Lyle Goodyear[†] Annika Younge[‡]

July 15, 2026

Abstract

We introduce a novel graph-theoretic model of risk-sharing and temporary migration in social networks with the presence of correlated income shocks. We characterize the conditions for incentive-compatible migration and risk-sharing arrangements, identifying how network structure directly influences agent decisions. Specifically, we prove that for unit-capacity networks, the set of households choosing not to migrate in the minimal stable arrangement coincides precisely with the network's k-core, where k depends explicitly on the severity of the correlated shocks and deterioration of relationships due to migration. We complement our theoretical results with numerical simulations on an Erdős–Rényi graph, illustrating how lesser-connected households are more likely to migrate, particularly as the shock becomes more severe.

Keywords: Social Networks, Informal Risk-Sharing, Temporary Migration

*Final paper proposal for ECON 291: Social and Economic Networks, Winter 2025

[†]Stanford University
[‡]Stanford University

1 Introduction

In developing countries, rural households face extreme uncertainty in year-to-year income. While income volatility varies household-to-household, a village may also experience negative shocks that affect all residents, i.e., adverse weather or the spread of disease. In response to uncertainty, rural communities frequently engage in informal risk-sharing, wherein households make monetary transfers to their adversely-affected neighbors with the expectation that the favor will be repaid in the future (Morten [2019]). In addition to village-endogenous risk-sharing, households may also self-insure by sending a member to temporarily migrate and earn a more stable income. While abroad, temporary migrants send remittance payments to their household in the village, which may in turn supplement risk-sharing within the village. The interplay between correlated income shocks, informal risk-sharing, and temporary migration is incredibly rich and warrants investigation.

To that end, we propose a novel[1] graph-theoretic model of temporary migration and risk-sharing in response to correlated income shocks. We first introduce the model and characterize the conditions under which a transfer arrangement and set of migration decisions is incentive compatible. We then analyze the convenient case where all relationships in the network are of equal strength, demonstrating that the set of households who choose not to send a migrant in the migrant minimal stable arrangement is exactly the k-core of the graph, where k is a function of the severity of the global shock and the amount by which migrants' relationships with the village deteriorate. Finally, we implement a numerical simulation on an Erdős–Rényi graph to illustrate our findings.

While we believe that the initial research presented in this proposal is well-considered, our primary aim is to provide justification for the long-term pursuit of this project. As such, we will hint at possible future results and current shortcomings wherever possible.

[1] To our knowledge.

2 Model

Our model draws significant inspiration from (Ambrus et al. [2014]). We consider a finite, undirected graph $G = (V, E)$ with $|V| = n$ agents and $|E| = m$ edges. Agents may represent individuals or households; in the former interpretation, individuals choose to temporarily migrate to insure themselves and pay remittance to their friends, while in the latter interpretation households elect to send a migrant member who will supplement their income. Links between agents represent personal or business relationships. We assume that the network is exogenously given and study the patterns of migration and risk-sharing that arise on it.

Each edge $e = (i, j) \in E$ has a corresponding capacity $c_{ij} \in \mathbb{R}_{>0}$. We denote the vector of capacities as $c \in \mathbb{R}^m_{>0}$. We may interpret the capacity c_{ij} of the relationship between agents i and j as the utility that each agent derives from their relationship. The utility c_{ij} could be a direct result of anticipated economic interaction between i and j, or it could represent a monetary encoding of the strength of the agents' friendship. Since the graph is undirected, we assume that $c_{ij} = c_{ji}$, and we may use (i, j) and (j, i) interchangeably to denote the edge between i and j.

Agents in the network wish to insure themselves against adverse shocks to their income. Formally, each agent faces uncertainty in their income, or *endowment*. An agent's endowment consists of two components: an idiosyncratic component e_i, drawn independently for each agents from a commonly known joint distribution, and a global shock $\theta \geq 0$ drawn from a commonly known distribution[2]. The shock θ initially affects all agents in the network—that is, agent i's endowment is given by $e_i - \theta$. The vector of endowments e and the shock θ are observed by all agents.

In response to the shock θ, agents may choose to temporarily migrate. If an individual i migrates, she does not experience the shock and her endowment becomes e_i. However,

[2]For the purposes of this paper, we assume θ is exogenously given.

we assume that migration impacts the strength of her friendships: for each $j \in N(i)$[3], the capacity of link (i, j) is reduced to λc_{ij} for some exogenously given constant $0 \leq \lambda < 1$. The *migration decision* of agent i is denoted by $d_i \in \{0, 1\}$ where $d \in \{0, 1\}^n$ specifies the migration decisions of all agents in the network. We let ϕ^d_{ij} denote the *effective capacity* of edge (i, j), where

$$\phi^d_{ij} = \begin{cases} \lambda c_{ij} \text{ if either of } d_i, d_j = 1 \\ c_{ij} \text{ if both } d_i, d_j = 0 \end{cases} \tag{1}$$

That is, the capacity of edge (i, j) is reduced to λc_{ij} if *either* of i or j chooses to migrate. For simplicity, we overload notation and let $\phi^d_{ij} : \{0, 1\} \to \mathbb{R}_{>0}$ denote the effective capacity of link (i, j) given a choice of d_i (fixing d_j). In other words, $\phi^d_{ij}(0) = \lambda c_{ij}$ if and only if $d_j = 1$; likewise $\phi^d_{ij}(1) = \lambda c_{ij}$ regardless of d_j.

To insure against uncertainty in endowments, agents participate in informal risk-sharing. Formally, a *risk-sharing arrangement* is a collection of payments made between agents given a specific realization of endowments. We denote the arrangement with t, where t_{ij} is the net amount transferred from i to j; we thus assume that $t_{ij} = -t_{ji}$. In addition to relationships, agents derive utility from consumption. The set of transfers t implements a consumption vector x, where

$$x_i = e_i - \theta^{d_i} - \sum_{j \in N(i)} t_{ij} \tag{2}$$

Finally, an agent's full utility is given by their consumption and the effective value of links given the vector of migration decisions. Formally, agent i's utility is given by

$$U_i(x_i + \sum_{j \in N(i)} \phi_{ij}) \tag{3}$$

where U_i is a strictly increasing, concave function. We assume that consumption and friendship are perfect substitutes, thus the argument to U_i is their sum.

[3] $N(i)$ denotes the neighborhood of i, the set of nodes that share an edge with i.

2.1 Incentive Compatibility

The primary goal of this paper will be to understand the conditions under which a set of transfers and migration decisions is stable—as well as which joint transfer and migration arrangements are induced by specific network structures. We say that a joint arrangement is *incentive compatible* if no agent would derive strictly higher utility by deviating from the arrangement, fixing the decisions of other agents. We may imagine that a central mechanism computes a joint arrangement given the network, endowment realizations, and global shock, and every agent observes the arrangement and decides whether to deviate. In our model, we allow for the following set of deviations. An agent may (1) withhold payment from one or more of its neighbors and in turn sever the link between them. They may also (2) update their migration decision, or (3) update their migration decisions *and* sever links with any number of their neighbors. To demonstrate that a joint arrangement is incentive compatible, it is sufficient to show that no agent benefits from (1) or (2) since their effects on utility are additive.

Deviation by (2) is not profitable when $t_{ij} \leq \phi_{ij}$ for all pairs $i \neq j$. This is a direct implication of the assumption that consumption and friendship are perfect substitutes. For migrating agents (such that $d_i = 1$), deviation by (1) is not profitable when

$$\theta > \sum_{j \in N(i)} \phi_{ij}(0) - \sum_{j \in N(i)} \phi_{ij}(1) \tag{4}$$

In other words, migrants will not have an incentive to change their decision if the *loss* of the shock θ (which they will now experience) outweighs the *gain* in friendship value from changing decisions. Similarly, for non-migrating agents (such that $d_i = 0$), incentive compatibility holds when

$$\theta \leq \sum_{j \in N(i)} \phi_{ij}(0) - \sum_{j \in N(i)} \phi_{ij}(1) \tag{5}$$

In other words, the *loss* in friendship value from changing decisions must outweigh the *gain* of θ (since the agent would no longer experience the global shock). We make the assumption that transfers are not re-calculated when an agent changes their migration decision to simplify analysis. Furthermore, we assume that if an agent is otherwise indifferent between staying or migrating, then they *strictly* prefer staying; thus (4) is a strict inequality.

2.2 Network Structure and Incentive Compatibility

Observe that under our formulation, that migration decisions are *strategic complements*—specifically, the utility an agent derives from migrating is increasing in the number of her neighbors who choose to migrate. We therefore might expect i to be more likely to migrate in a stable arrangement if i's neighbors also choose to migrate. On the other hand, if i has a high weighted degree, the loss in friendship value from migrating will also be high. It follows that better-connected parts of the network—in terms of either number of neighbors or the strength of connections—will be less inclined to migrate in a stable arrangement. This reflects previous results in risk-sharing networks: (Ambrus et al. [2014]) find that well-connected subgraphs implement perfect risk-sharing, while agents outside of these subgraphs consume less in a stable transfer arrangement. We interpret this result as implying that risk-sharing in these subgraphs is good enough to discourage migration. Furthermore, lesser-connected parts of the graph will be more inclined to migrate, particularly if the shock θ is bad.

2.3 Unit Capacity Networks

To formalize these claims, we now restrict our attention to the analytically convenient case of unit capacity graphs (wherein $c_{ij} = 1$ for all $i \neq j$). In this case, the effective capacity of any link incident to a migrating agent is simply λ. Therefore, the difference in friendship value between regimes in which agent i does or doesn't migrate is

$$\sum_{j \in N(i)} \phi_{ij}(0) - \sum_{j \in N(i)} \phi_{ij}(1) = (\sum_{j \in N(i):d_j=1} \lambda + \sum_{j \in N(i):d_j=0} 1) - (\sum_{j \in N(i):d_j=1} \lambda + \sum_{j \in N(i):d_j=0} \lambda) \quad (6)$$

$$= (1 - \lambda)k_i \tag{7}$$

where we define $k_i = |\{j \in N(i) : d_j = 0\}|$ to be the number of agents in the neighborhood of i who are not migrating. Since agents strictly prefer to stay when they are otherwise indifferent between migration decisions, we derive the following threshold condition for when an agent will decide to stay under unit capacities:

$$(1 - \lambda)k_i \geq \theta \implies k_i \geq \frac{\theta}{1 - \lambda} \tag{8}$$

In other words, an agent will be incentivized to stay in any stable arrangement if and only if the number of its neighbors who stay is at least the threshold number $\frac{\theta}{1-\lambda}$. Note that if $\frac{\theta}{1-\lambda} < 1$, then for any connected graph, there are always at least two stable arrangements: one in which no agent migrates, and one in which every agent migrates.

For the purposes of this paper, we will compare joint transfer and migration arrangements only by differences in their migration vectors, since our primary focus is on incentive compatibility for migration-based deviations (which does not rely on the transfer arrangement). Define a partial ordering (by migration decision) of stable joint arrangements as follows. Given two migration arrangements d and d', we say that d' is "greater than" d if for $D^d = \{i \in V : d_i = 0\}$ (the set of non-migrating agents in d) we have $D^d \subset D^{d'}$; similarly, d' is "greater than or equal to" d if $D^d \subseteq D^{d'}$. This allows us to define a *migrant minimal* stable arrangement d_* such that $D^d \subseteq D^{d_*}$ for any migration vector d.

Theorem 1 *For a unit capacity graph G, the set of non-migrating agents in the unique migrant minimal stable arrangement d_* is exactly the $\frac{\theta}{1-\lambda}$-core of G.*

Proof The proof of Theorem 1 is inspired by a similar result from (Gagnon and Goyal [2017]). Recall that the k-core of a graph G is the unique maximal subgraph in which every node has at least k neighbors in the subgraph (Seidman [1983]). To prove Theorem 1, we first demonstrate necessity: that is, in any stable arrangement d, every agent i with $d_i = 0$

has at least $\frac{\theta}{1-\lambda}$ neighbors who also do not migrate, so the set D^d induces a subgraph of G in which every vertex has degree at least $\frac{\theta}{1-\lambda}$. This is directly implied by the incentive compatibility constraints. To prove sufficiency, we must show that every vertex in the $\frac{\theta}{1-\lambda}$-core of G is also contained in D^{d_*}. To see this, note that every node i in the $\frac{\theta}{1-\lambda}$-core of G has at least $\frac{\theta}{1-\lambda}$ neighbors which are also in the $\frac{\theta}{1-\lambda}$-core. If we construct a migration vector d such that $d_i = 0$ if and only if i is in the $\frac{\theta}{1-\lambda}$-core, then no agent would have an incentive to deviate and we obtain a stable arrangement d such that $D^d \subseteq D^{d_*}$. This concludes the proof of Theorem 1.

Theorem 1 allows us to directly apply structural results regarding k-cores to the set of non-migrating agents. We suspect there is a very rich set of structural results that directly apply to our setting, but for the purposes of this proposal, we only consider the following: the k-cores of a graph are uniquely defined and nested such that $k+1$-core is always a subgraph of the k-core. In our setting, this implies that as $\frac{\theta}{1-\lambda}$ increases, we "peel" away agents with lower degree from the set of agents who do not migrate (in the migrant minimal stable arrangement). The last agents to migrate will therefore be the agents with highest degree. Furthermore, the highest-degree agents will migrate in d_* exactly when $\frac{\theta}{1-\lambda} < 1$— since as stated earlier, the migrant *minimal* arrangement when $\frac{\theta}{1-\lambda} < 1$ is such that no agent migrates.

3 Simulation

To further characterize the relationship between migration decisions and network structure, we implement a numerical simulation, aiming to visualize the "peeling" away of lower-degree nodes from the set of non-migrating agents as $\frac{\theta}{1-\lambda}$ increases. We consider a fixed sample of an Erdős–Rényi graph with 50 nodes where links form with probability $p = 1/2$. The capacity of each link is set to 1. All agents share the utility function $U_i = \log(x_i + c_i)$ where c_i is the sum of the effective capacities of i's edges. Finally, each idiosyncratic endowment

e_i is drawn independently from the uniform distribution on $(0, 1)$.

We solve for a joint transfer and migration arrangement by initially randomizing the migration decision vector. We then solve a planner's problem which maximizes the sum of agent utilities while respecting the effective capacity constraints. Once we have computed the consumption of each agent, we check whether they have incentive to deviate and flip their migration decision if they will benefit from deviating. We repeat this process iteratively until no agent has an incentive to deviate and we achieve a stable joint arrangement.

We choose to fix $\lambda = 0.5$ and vary θ. Figures 1-5 display the resulting migration decisions in the network, where green nodes signify non-migrants and red nodes signify migrants. Below each figure we report the value of $\frac{\theta}{1-\lambda}$. The network is visualized using a spring algorithm which results in higher-degree nodes displaying closer to the visual center of the graph. As expected, we observe that increasing $\frac{\theta}{1-\lambda}$ results in lower-degree agents "peeling" away from the edges of the network by migrating.

In Figures 6-10, we color nodes according to their consumption, with higher consuming nodes appearing more blue. This visualization gives us a rough idea of the interplay between migration and risk-sharing—we generally observe that migrants actually achieve *higher* consumption by self-insuring against the shock θ than non-migrants are able to achieve purely from transfers. We aim to investigate this interplay further in the full paper.

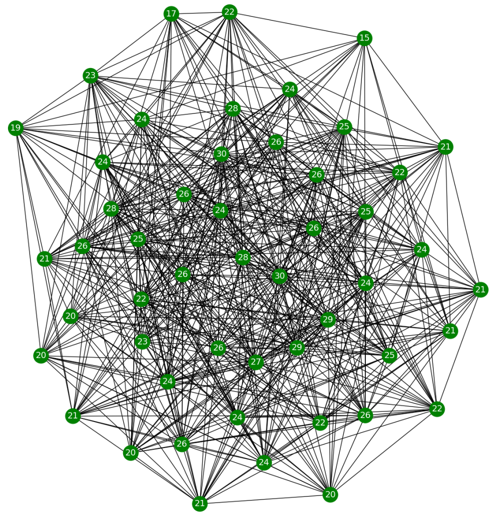

Figure 1: $\frac{\theta}{1-\lambda} = 14.7$

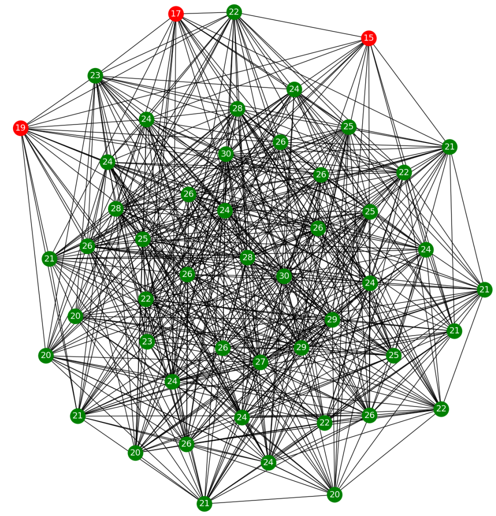

Figure 2: $\frac{\theta}{1-\lambda} = 19.6$

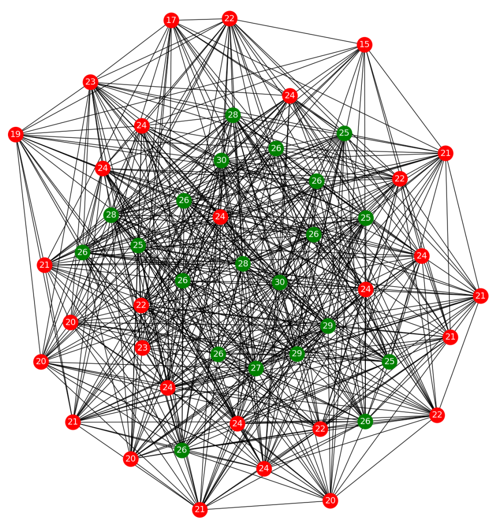

Figure 3: $\frac{\theta}{1-\lambda} = 24.5$

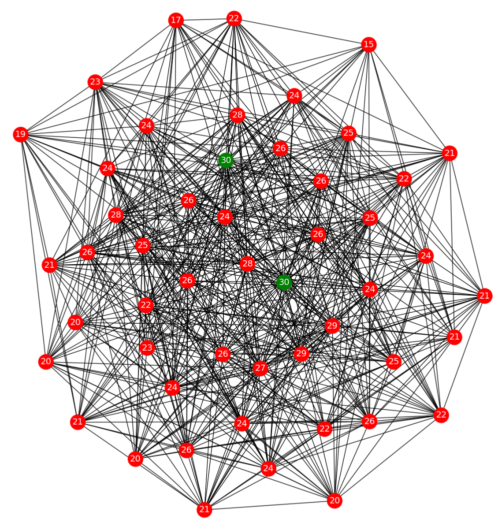

Figure 4: $\frac{\theta}{1-\lambda} = 29.4$

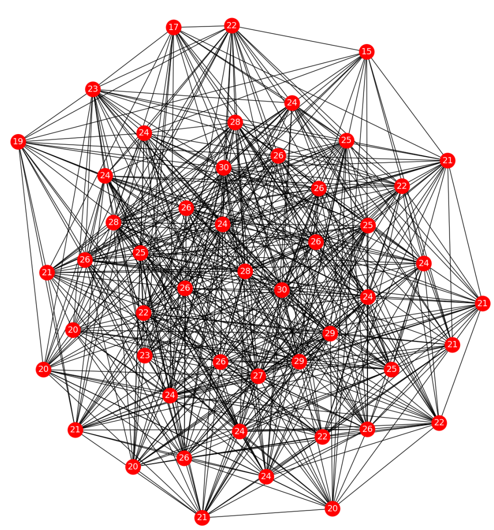

Figure 5: $\frac{\theta}{1-\lambda} = 34.3$

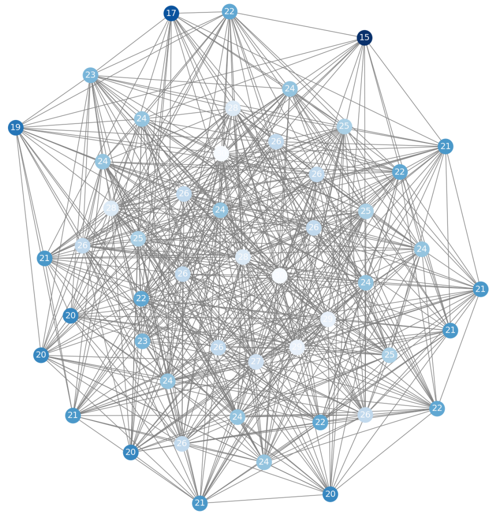

Figure 6: $\frac{\theta}{1-\lambda} = 14.7$

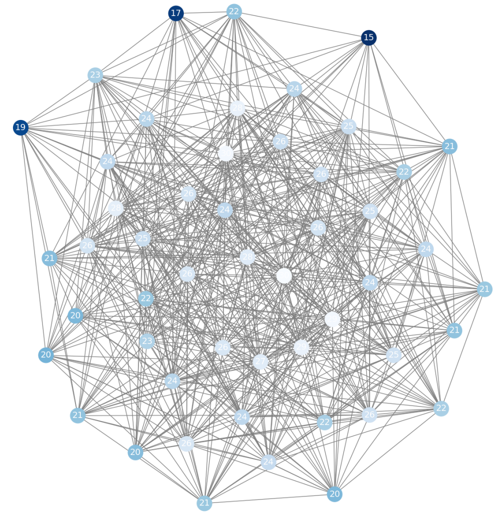

Figure 7: $\frac{\theta}{1-\lambda} = 19.6$

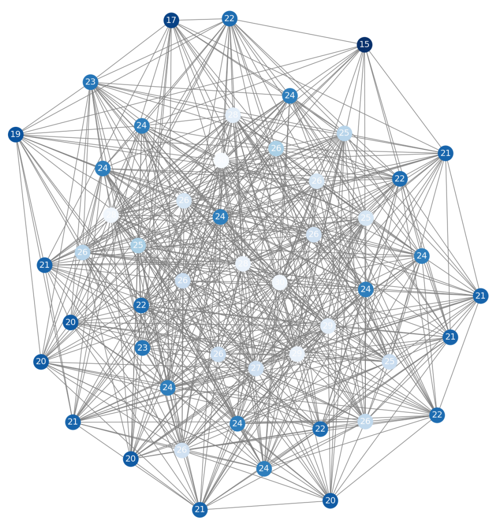

Figure 8: $\frac{\theta}{1-\lambda} = 24.5$

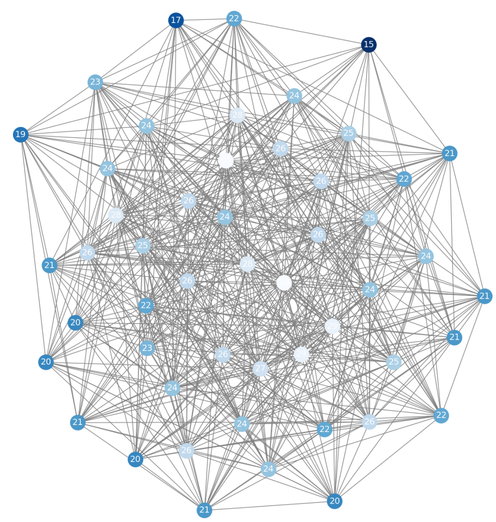

Figure 9: $\frac{\theta}{1-\lambda} = 29.4$

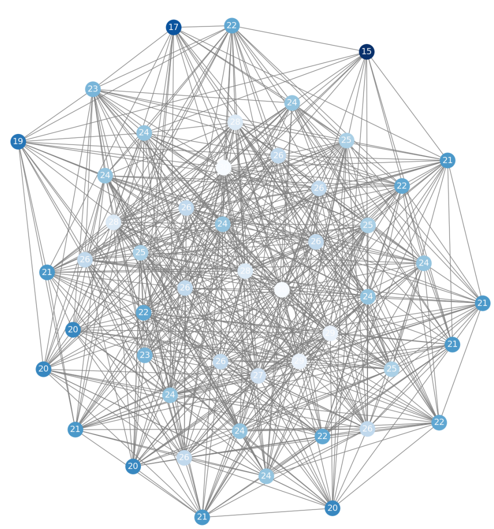

Figure 10: $\frac{\theta}{1-\lambda} = 34.3$

4 Future Work

Given that this is a proposal, we would like to outline possible future directions for this line of research.

4.1 Uniqueness and Generalization to Heterogeneous Capacities

While we hinted at settings where multiple stable migration arrangements exist, we would like to fully characterize the multiplicity of stable arrangements under various assumptions. For instance, for $\frac{\theta}{1-\lambda} \geq 1$, is the migrant minimal stable arrangement also the unique arrangement with respect to this migration vector? We suspect there may exist multiple stable arrangements where different parts of the graph choose to migrate, particularly with heterogeneous capacities. On that note, we would like to generalize our results for unit capacity graphs to broader settings where capacities are heterogeneous.

4.2 Coalitional Migration

Currently in our model, we assume that links deteriorate between all neighbors of a migrant. However, there is evidence that migrants often travel together and maintain relationships among themselves in their location (Munshi [2003]). We could imagine extending the model to allow for coalitional migration wherein any subset of agents can elect to migrate together and maintain the full capacity of their links. We would need to make careful assumptions to ensure that there is not an incentive for the entire network to migrate (as there would be in our current model). This could be accomplished by introducing income uncertainty at destination as well, which would also better capture conditions for real-life migrants. There are tradeoffs to this approach, however, particularly in the model's tractability.

4.3 Closer Analysis of Risk-Sharing

While risk-sharing is a prominent feature of our model, we do little to analyze how network structure jointly affects the quality of risk-sharing and the migration decisions. In the full paper, we would like to carefully consider how to formalize the initial interaction effects observed in Figures 6-10. In particular, we would investigate whether the risk-sharing "islands" observed by (Ambrus et al. [2014]) also appear in this model. Furthermore, do risk-sharing islands correlate with migration islands, or are they distinct objects?

4.4 Endogenous Link Formation and Repeated Interaction

Our model treats links and capacities as exogenous, but we could imagine extending to a setting where the network structure is also determined exogenously. Similar to (Ambrus et al. [2014]), we could consider having capacities determined strategically with the anticipation of future transfers. We could also consider repeated interactions in which agents make transfers and temporarily migrate in a stage game. Here, agents need to carefully consider whether defaulting on payment is worth losing the transfer potential of that link for the rest of the game. However, this would also significantly increase the complexity of the model.

4.5 Empirical Work

A large potential avenue for this line of research is to study migration and risk-sharing in real-world networks. Similar to (Ambrus et al. [2014]), we could calibrate our model to data, confirming whether our results pertaining to network structure arise in real-world networks with those structural properties. Given the wealth of studies examining risk-sharing in real networks, we could likely use preexisting data. However, we would also need access to data on migration patterns in the network which may not be readily available. By comparing our model's predictions to real-world patterns of migration and risk-sharing, we may uncover subtleties that we failed to account for.

References

Attila Ambrus, Markus Mobius, and Adam Szeidl. Consumption Risk-Sharing in Social Networks. *American Economic Review*, 104(1):149–182, January 2014. ISSN 0002-8282. doi: 10.1257/aer.104.1.149. URL https://www.aeaweb.org/articles?id=10.1257/aer.104.1.149.

Julien Gagnon and Sanjeev Goyal. Networks, Markets, and Inequality. *American Economic Review*, 107(1):1–30, January 2017. ISSN 0002-8282. doi: 10.1257/aer.20150635. URL https://www.aeaweb.org/articles?id=10.1257/aer.20150635.

Melanie Morten. Temporary Migration and Endogenous Risk Sharing in Village India. *Journal of Political Economy*, 127(1):1–46, February 2019. ISSN 0022-3808. doi: 10.1086/700763. URL https://www.journals.uchicago.edu/doi/abs/10.1086/700763. Publisher: The University of Chicago Press.

Kaivan Munshi. Networks in the Modern Economy: Mexican Migrants in the U. S. Labor Market*. *The Quarterly Journal of Economics*, 118(2):549–599, May 2003. ISSN 0033-5533. doi: 10.1162/003355303321675455. URL https://doi.org/10.1162/003355303321675455.

Stephen B. Seidman. Network structure and minimum degree. *Social Networks*, 5(3):269–287, September 1983. ISSN 0378-8733. doi: 10.1016/0378-8733(83)90028-X. URL https://www.sciencedirect.com/science/article/pii/037887338390028X.